Localised solutions in Magnetoconvection

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Within the dark central umbra of sunspots we can observe localised bright convective features, called umbral dots. It is believed that sunspots, and therefore umbral dots, are formed by the interactions between convection and strong magnetic fields. Numerically localised convective features have been observed in two-dimensional, and highly truncated three-dimensional, simulations of convection in the presence of an imposed magnetic field.

The focus of this dissertation is to consider convection in the presence of a strong magnetic field, as present in the central umbra of a sunspot, to gain an understanding of the formation and structure of localised convective patterns.

We begin with a brief survey of solar observational results and previous results from mathematical modelling of the system. We then reconsider the two-dimensional problem through the formation of discrete models that allow very large systems to be considered. Results from the discrete models agree well with results from the full system. In Chapter 3, the numerical methods used to model magnetoconvection are described and full details of the mathematical problem are given. A study of compressible convection in the presence of a strong imposed magnetic field, in three dimensions is then completed and the results presented. To allow large domains to be investigated a Swift-Hohenberg type model for Boussinesq magnetoconvection is developed in Chapter 5. The thermal conductivity is allowed to vary with depth. This vertical variation breaks the up-down symmetry of Boussinesq convection and allows hexagonal patterns to form.

It is found that while localised convective regions can be found in a reasonably generic way for the two-dimensional problem, they typically appear only as transients in the three-dimensional problem. Instead we find relatively weak convection patterns whose amplitude varies on a long length-scale.

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